Multiple Choice

1. A fair die is rolled 60 times, and the value of $\chi^2$ is computed using expected counts of 10 for each face. If this is repeated many times, the shape of the distribution of the values of $\chi^2$ should be
   A. approximately normal.
   B. skewed right.
   C. skewed left.
   D. uniform.
   E. bimodal.

2. The major difference between the chi-square test of homogeneity and the chi-square test of independence is the
   A. number of categories.
   B. sample size.
   C. method of sampling.
   D. size of the $\chi^2$ statistic.
   E. number of degrees of freedom.

3. Which of these statements is not true?
   A. A chi-square test of independence that is statistically significant shows a cause-and-effect relationship.
   B. A segmented bar chart is useful in observing when two variables might be associated.
   C. As the number of categories increases, the $\chi^2$ distribution approaches the normal distribution.
   D. The chi-square tests involve categorical variables.
   E. The chi-square goodness-of-fit test is an extension of the z-test to more than two categories.

4. The null hypothesis is rejected in a chi-square test of significance when
   A. the test conditions are satisfied.
   B. the $P$-value is larger than $\alpha$, the level of significance.
   C. the $P$-value is larger than $1 - \alpha$.
   D. the $\chi^2$ statistic is smaller than the critical value for the given level of significance.
   E. the $\chi^2$ statistic is larger than the critical value for the given level of significance.

5. Which test is appropriate for determining whether a random-digit generator is truly random in terms of the proportions of each digit it produces?
   A. the chi-square goodness-of-fit test
   B. the chi-square test of homogeneity
   C. the chi-square test of independence
   D. Either B or C is appropriate.
   E. None of these tests is appropriate.
6. Suppose that 85% of all Americans are right-handed, 10% are left-handed, and 5% are ambidextrous. A random sample of 114 Mississippians includes 80 right-handers, 31 left-handers, and 3 ambidextrous. What is the value of the \( \chi^2 \) statistic for the goodness-of-fit test that the distribution of handedness for all Mississippians is the same as the distribution for all Americans, and what is the correct conclusion?

A. \( \chi^2 = 37.92; \) not significant at the 0.05 level
B. \( \chi^2 = 37.92; \) significant at the 0.05 level
C. \( \chi^2 = 2.37; \) not significant at the 0.05 level
D. \( \chi^2 = 2.37; \) significant at the 0.05 level
E. Because the conditions are not satisfied, significance can’t be determined.

7. An educator randomly selects 300 statistics students to check whether there is a relationship between a student passing the course and his or her number of absences. These results were reported.

<table>
<thead>
<tr>
<th>Number of Classes Missed</th>
<th>0–1</th>
<th>2–3</th>
<th>4 or More</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passed Statistics Course</td>
<td>30</td>
<td>50</td>
<td>100</td>
<td>180</td>
</tr>
<tr>
<td>Didn’t Pass Statistics Course</td>
<td>10</td>
<td>30</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>80</td>
<td>180</td>
<td>300</td>
</tr>
</tbody>
</table>

For a chi-square test of independence, what type of error may have been made?

A. Type I: the \( P \)-value was greater than 0.05.
B. Type I: the \( P \)-value was less than 0.05.
C. Type II: the \( P \)-value was greater than 0.05.
D. Type II: the \( P \)-value was less than 0.05.
E. Either type of error is possible.

Short Answer

8. In Question 7, what is the expected count for the number of students who were absent 0 or 1 time and who also passed the statistics course? Show your computation.

\[
\frac{40 \times 180}{300} = 24
\]
Matching

9. Match the survey designs in parts a–c with the most appropriate chi-square test: goodness of fit, homogeneity, or independence.

a. You are told that the number of cracked M&M’s depends on color. To check this claim, you randomly select 100 M&M’s and sort by color and whether the M&M is cracked or uncracked.

   a. chi-square test of independence

b. You are told that the distribution of M&M’s colors is as follows: 13% red, 16% green, 14% yellow, 20% orange, 24% blue and 13% brown. To check this claim, you randomly select a sample of M&M’s and count the number of M&M’s of each color.

   b. chi-square test of goodness of fit

c. You are told that the number of cracked M&M’s depends on color. To check this claim, you randomly select 100 M&M’s of each color and count the number of cracked and uncracked M&M’s of each color.

   c. chi-square test of homogeneity
Open Ended
10. A sociology class is interested in knowing whether there is a relationship between personality style and type of extracurricular activity. The class took a random sample of 200 students from a large high school and recorded these data.

<table>
<thead>
<tr>
<th>Personality Style</th>
<th>Extrovert</th>
<th>Normal</th>
<th>Introvert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Extracurricular Activity</td>
<td>Sports</td>
<td>21</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Club</td>
<td>16</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>None</td>
<td>3</td>
<td>16</td>
</tr>
</tbody>
</table>

Using the standard four-step process (name test and check conditions; state hypotheses; compute test statistic and $P$-value and draw a sketch; and write a conclusion in context), test the null hypothesis that personality style and type of extracurricular activity are independent.

Name the test. Chi-square test of independence.

Check conditions. A random sample was taken from one large population. Each outcome can be classified into one of three categories for each of the two variables. The expected number in each cell is at least 5. The table of expected counts is shown here.

<table>
<thead>
<tr>
<th>Personality Style</th>
<th>Extrovert</th>
<th>Normal</th>
<th>Introvert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Extracurricular Activity</td>
<td>Sports</td>
<td>18</td>
<td>41.85</td>
</tr>
<tr>
<td></td>
<td>Club</td>
<td>13.8</td>
<td>32.085</td>
</tr>
<tr>
<td></td>
<td>None</td>
<td>8.2</td>
<td>19.065</td>
</tr>
</tbody>
</table>

State your hypotheses.

$H_0$: Personality style and activity type are independent in the population of all students in this high school.

$H_a$: Personality style and activity type are not independent.

Compute the test statistic and $P$-value and draw a sketch.

$$\chi^2 = \sum \frac{(O - E)^2}{E} = 11.05$$

$P$-value $= 0.026$ ; 4 degrees of freedom

Write a conclusion in context. Because the $P$-value of 0.026 is less than 0.05, the null hypothesis that the two variables are independent can be rejected. In other words, there is evidence that there is an association between personality style and activity type.
11. A pollster was hired by a television studio to take a random sample of 400 high school students, 100 from each of grades 9–12, and ask the students whether they watch at least one reality television show on a regular basis. The percentages that replied yes for grades 9, 10, 11, and 12 were 28%, 18%, 30%, 33%, respectively. The studio wishes to know whether there is evidence that the percentage varies among the grade levels.

a. What significance test would you use for this design—a test of goodness of fit, homogeneity, or independence? Explain your choice.

A chi-square test of homogeneity because four independent random samples were taken from four large populations.

b. Perform the test you selected, showing all four steps.

**Name the test.** Chi-square test of homogeneity.

**Check conditions.** Independent simple random samples of fixed sizes (100) were taken from four large populations. Each outcome falls into exactly one of two categories (which are the same in all four populations). The expected frequency is at least 5 in each cell. The table of expected counts is shown here.

<table>
<thead>
<tr>
<th>Grade</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>27.25</td>
<td>27.25</td>
<td>27.25</td>
<td>27.25</td>
<td>109</td>
</tr>
<tr>
<td>No</td>
<td>72.75</td>
<td>72.75</td>
<td>72.75</td>
<td>72.75</td>
<td>291</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>400</td>
</tr>
</tbody>
</table>

**State your hypotheses.**

H₀: The proportion of all high school students who would reply yes (watch a reality show) is the same for each grade level.

H₁: At least one grade level has a proportion of yes responses that is different from the proportion in another grade level.

**Compute the test statistic and P-value.**

\[ \chi^2 = \sum \frac{(O - E)^2}{E} = 6.39 \]

P-value = 0.094; 3 degrees of freedom

**Write a conclusion in context.** Because the P-value is larger than 0.05, there is not enough evidence (at the 0.05 significance level) to reject the null hypothesis that the proportion of all high school students who would reply yes (watch a reality TV show) is the same for the four grade levels.