Probability

Part 1: Basic Terms
Sample space

The collection of all possible outcomes of a probability experiment

Example: Roll a die
S=\{1,2,3,4,5,6\}
Event

Any collection of outcomes from the sample space

Example 1:
Rolling a prime #:
E = \{2,3,5\}

Example 2:
Rolling a prime # or even number:
E = \{2,3,4,5,6\}
Complement

Consists of all outcomes that are **not** in the event

Example:
Not rolling a even #: 

\[ E^C = \{1,3,5\} \]
Union

The event A or B \((A \cup B)\) happening.

- consists of all outcomes that are in at least one of the two events

Example:
Rolling a prime # or even number

\[ E = \{2, 3, 4, 5, 6\} \]
Intersection

The event $A$ and $B$ ($A \cap B$) happening

• consists of all outcomes that are in both events

Example:

Drawing a red card and a “2”:

$E = \{2 \text{ hearts}, 2 \text{ diamonds}\}$
Mutually Exclusive (disjoint)

Two events have no outcomes in common.

Example:
Roll a “2” or a “5”
Venn Diagrams

- Used to display relationships between events
- Helpful in calculating probabilities
Venn diagram - Complement of A
Venn diagram - A or B
Venn diagram - A and B
Venn diagram - disjoint events
Statistics & Computer Science & not Calculus
Calculus or Computer Science
(Statistics or Computer Science) and not Calculus
Statistics and not (Computer Science or Calculus)
Statistics or Computer Science? 170
Statistics and Computer Science? 20
Statistics or (Computer Science and Calculus)?

90
(Statistics or Computer Science) and Calculus?

50
Probability
Part 2: Rules and Equations
Classical Probability

Denoted by \( P(\text{Event}) \)

\[
P(E) = \frac{\text{favorable outcomes}}{\text{total outcomes}}
\]

This method for calculating probabilities is only appropriate when the outcomes of the sample space are equally likely.
Empirical (Observed) Probability

The relative frequency at which a chance experiment occurs

Example:
Flip a fair coin 30 times & get 17 heads

\[
\frac{17}{30}
\]
Law of Large Numbers

As the number of repetitions of a probability experiment increases, the difference between the relative frequency of occurrence for an event and the true probability approaches zero.
Basic Rules of Probability

Rule 1. **Legitimate Values**
For any event \( E \),
\[ 0 \leq P(E) \leq 1 \]

Rule 2. **Sample space**
If \( S \) is the sample space,
\[ P(S) = 1 \]
Basic Rules of Probability

Rule 3. Complement

For any event E,

\[ P(E) + P(\text{not } E) = 1 \]
Basic Rules of Probability

Rule 4. Addition

If two events $E$ & $F$ are disjoint,

$$P(E \text{ or } F) = P(E) + P(F)$$

(General) If two events $E$ & $F$ are not disjoint,

$$P(E \text{ or } F) = P(E) + P(F) - P(E \& F)$$
Ex. 1) A large auto center sells cars made by many different manufacturers. Three of these are Honda, Nissan, and Toyota. (Note: these are not simple events since there are many types of each brand.) Suppose that $P(H) = .25$, $P(N) = .18$, $P(T) = .14$.

Are these disjoint events? \[ \text{yes} \]

$P(H \text{ or } N \text{ or } T) = .25 + .18 + .14 = .57$

$P(\text{not } (H \text{ or } N \text{ or } T)) = 1 - .57 = .43$
Ex. 2) Musical styles other than rock and pop are becoming more popular. A survey of college students finds that the probability they like country music is .40. The probability that they liked jazz is .30 and that they liked both is .10. What is the probability that they like country or jazz?

\[ P(C \text{ or } J) = .4 + .3 - .1 = .6 \]
Independent

Two events are independent if knowing that one will occur (or has occurred) does not change the probability that the other occurs.

- A randomly selected student is female - What is the probability she plays soccer for WSH?

  Independent

- A randomly selected student is female - What is the probability she plays football for WSH?

  Not independent
Rule 5. Multiplication

If two events A & B are independent,

\[ P(A \& B) = P(A) \cdot P(B) \]

General rule:

\[ P(A \& B) = P(A) \cdot P(B \mid A) \]
Ex. 3) A certain brand of light bulbs are defective five percent of the time. You randomly pick a package of two such bulbs off the shelf of a store. What is the probability that both bulbs are defective?

Can you assume they are independent?

\[ P(D \& D) = 0.05 \cdot 0.05 = 0.0025 \]
Ex. 4)

If \( P(A) = 0.45 \), \( P(B) = 0.35 \), and \( A \& B \) are independent, find \( P(A \text{ or } B) \).

Is \( A \& B \) disjoint? NO, independent events cannot be disjoint

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \& B)
\]

\[
P(A \text{ or } B) = .45 + .35 - .45(.35) = 0.6425
\]
Ex. 5) Suppose I will pick two cards from a standard deck without replacement. What is the probability that I select two spades?

Are the cards independent? NO

\[ P(A \& B) = P(A) \cdot P(B|A) \]

\[ P(\text{Spade} \& \text{Spade}) = \frac{1}{4} \cdot \frac{12}{51} = \frac{1}{17} \]
Basic Rules of Probability

Rule 6. At least one

The probability that at least one outcome happens is 1 minus the probability that no outcomes happen.

\[ P(\text{at least 1}) = 1 - P(\text{none}) \]
Ex. 6) A certain brand of light bulbs are defective five percent of the time. You randomly pick a package of two such bulbs off the shelf of a store.

What is the probability that at least one bulb is defective?

\[
P(\text{at least one}) = P(D \& D^C) \text{ or } P(D^C \& D) \text{ or } P(D \& D) \text{ or } 1 - P(D^C \& D^C)
\]

\[
= 0.0975
\]
Ex. 7) For a sales promotion the manufacturer places winning symbols under the caps of 10% of all Dr. Pepper bottles. You buy a six-pack. What is the probability that you win something?

\[
P(\text{at least one winning symbol}) = 1 - P(\text{no winning symbols})
\]

\[
1 - 0.9^6 = 0.4686
\]
Rule 7: Conditional Probability

A probability that takes into account a given condition

\[
P(B \mid A) = \frac{P(A \cap B)}{P(A)}
\]

\[
P(B \mid A) = \frac{\text{and}}{\text{given}}
\]
Ex. 6) In a recent study it was found that the probability that a randomly selected student is a girl is .51 and is a girl and plays sports is .10. If the student is female, what is the probability that she plays sports?

\[
P(S \mid F) = \frac{P(S \cap F)}{P(F)} = \frac{.1}{.51} = .1961
\]
Ex. 7) The probability that a randomly selected student plays sports if they are male is .31. What is the probability that the student is male and plays sports if the probability that they are male is .49?

\[
P(S | M) = \frac{P(S \cap M)}{P(M)}
\]

\[
.31 = \frac{x}{.49}
\]

\[
x = .1519
\]
Conditional Probability

Practice Problem 1:
Only 5% of male high school basketball, baseball, and football players go on to play at the college level. Of these, only 1.7% enters major league professional sports. Of the athletes that do not play college sports, only 0.1% enters major league professional sports. What is the probability that a high school athlete will play professional sports?

What is the probability that a high school athlete does not play college sports if he plays professional sports?
Practice Problem #2
Only 5% of male high school basketball, baseball, and football players go on to play at the college level. Of these, only 1.7% enters major league professional sports. Of the athletes that do not play college sports, only 0.1% enters major league professional sports. What is the probability that a high school athlete does not play college sports if he plays professional sports?
Practice Problem #3
Management has determined that customers return 12% of the items assembled by inexperienced employees, whereas only 3% of the items assembled by experienced employees are returned. Due to turnover and absenteeism at an assembly plant, inexperienced employees assemble 20% of the items. Construct a tree diagram or a chart for this data.

What is the probability that an item is returned?
If an item is returned, what is the probability that an inexperienced employee assembled it?